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13. ABSTRACT (Maximum 200 words) Prof. R. Krishnamurti and Dr H. Yang have completed their investigation of finite amplitude Hele-Shaw convection with an imposed shear. Their interests were also to determine the effect of convective transport of momentum upon the mean flow. Prof. S. Blumsack has continued his analysis of approximate solutions of the barotropic vorticity equation subject to periodic boundary conditions. In another study, Prof. R. L. Pfeffer and student Wen Ding have made considerable progress solving the nonlinear steady state barotropic vorticity equation for parameters which correspond to a series of laboratory experiments involving forced flow of a rotating fluid over bottom topography. In these experiments, conducted in our Institute, the fluid is contained in a rotating circular cylindrical annulus with a differentially rotating, rigid, radially sloping, lid in contact with the top surface of the fluid. The fluid is forced into motion by the rotation of the lid. In another effort, Professors Barcilon and Blumsack and Mr. Y. Chang are investigating the dynamics of equilibrium states in a sheared barotropic channel flow. This flow is relevant to meteorological flows and the object of this work is to understand the population of certain states, called free modes and comprehend how the actual flow in phase space is attracted from one state to another. The present research effort has investigated

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new parallel computing techniques and fast algorithms for the efficient utilization of parallel computers in the solution of the shallow water equations. Progress has been made toward understanding the development and nonlinear behavior of traveling baroclinic waves in the atmosphere and their interaction with topographically and thermally forced planetary waves.

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Nonlinear Dynamics Underlying Atmospheric Predictability

by

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INTRODUCTION

The second year of this University Research Initiative has been even more productive than the first. To date, 6 papers have been published or accepted for publication and 2 have been submitted for publication based on full or partial support of AFOSR 89-0462 as follows:

- A study of Baroclinic Wave Behavior over Bottom Topography Using Complex Principal Component Analysis of Experimental Data, by R. L. Pfeffer, J. Ahlquist, R. J. Kung, Y. Chang and G. Li; *J. Atmos. Sci.*, **47**, 67-81, 1990.
- Effects of Wave-Wave and Wave-Mean Flow Interactions on the Evolution of a Baroclinic Wave, by A. Barcilon and T. Nathan; *Geophys. Astrophys. Fluid Dyn.*, **56**, 59-79, 1991.
 - Asymmetric Ekman Dissipation, Sloping Boundaries and Linear Baroclinic Instability, by H. Weng and A. Barcilon; *Geophys. Astrophys. Fluid Dyn.*, in press, 1991.
 - Reflection of Hydrostatic Mountain Waves from Spatially Nonuniform layers, by W. Blumen and A. Barcilon; *Geophys. Astrophys. Fluid Dyn.*, in press, 1991.
 - A Comparison of the Impact of Two Time-Differencing Schemes on the NASA/GLAS Climate Model, by R. L. Pfeffer, I. M. Navon and X. Zou; Accepted for publication in *Mon. Wea. Rev.*, 1991.
 - EXSHALL: A Turkel-Zwas Explicit Large Time-Step FORTRAN Program for Solving the Shallow-Water Equations in Spherical Coordinates by I. M. Navon and Jian Yu, Accepted for publication in *Computers in Geoscience*.
- Hele Shaw Convection with Imposed Shear, Part I, by R. Krishnamurti and H. Yang; submitted to *J. Fluid Mech.*, 1991.
- Hele Shaw Convection with Imposed Shear, Part II, by H. Yang and R. Krishnamurti; to be submitted to *J. Fluid Mech.*, 1991.

The following sections present a summary of progress on research under the grant, including the work of four Ph.D. candidates (one of whom received his Ph.D. degree within the last year).

RAYLEIGH-BÉNARD CONVECTION

Thermal convection in a horizontal layer heated from below ("Bénard convection"), in the turbulent range, sometimes is accompanied by a large scale mean shear flow, so that not only heat but also momentum is transferred across the layer (R. Krishnamurti and L. N. Howard, *Proc. Nat. Acad. Sci.* **78**, 1981-1985, 1981). This means shear flow appears to arise spontaneously, in a manner reminiscent of a bifurcation, though it occurs when the fluid is already in turbulent

motion. In a recent study, Prof. L. Howard (*Studies in Applied Math.*, 83, 273–285, 1990) obtained bounds on the magnitudes of the heat and momentum fluxes (in terms of the Rayleigh number, which measures the strength of the thermal driving) by an extension of methods formerly used to estimate the heat flux in the absence of large scale flow (L. Howard, *J. Fluid Mech.*, 17, 405–432, 1963).

These last results have now been extended further, to the case in which there is also a horizontal translation of one of the boundary plates with respect to the other, i.e., boundary conditions which in the absence of any turbulence would be consistent with a plane Couette flow superimposed on thermal convection. Of course, a large scale shear flow and associated momentum transfer is to be expected when there is mechanical driving alone (turbulent plane Couette flow), but its occurrence with thermal driving alone suggests that the situation with both present may also have some surprises—in particular the momentum flux may be enhanced significantly by the presence of convection.

In this case there are two external “driving” parameters, the Rayleigh number and a Reynolds number based on the velocity of the moving plate. The estimate then takes the form of the description of a finite region in the plane of heat flux and momentum flux, depending on the Rayleigh and Reynolds numbers, within which any realized values of these fluxes must lie. These results have been presented at the Miles Symposium at Scripps Institute of Oceanography; publication is expected in the near future.

The estimates obtained depend on the solution of a rather complicated variational problem which has so far not in fact been exactly solved, but only itself bounded in terms of a somewhat simpler variational problem that Dr. Howard had solved before (*loc. cit.*). It was possible to solve that problem analytically in terms of elliptic integrals, but a similar analytical solution of the more complicated one has eluded him; a numerical study is probably feasible and is under consideration. (Indeed, a significant amount of numerical work was required in adapting the old elliptic integral solution to the bound of the more complicated variational problem.) The difficulty is not so much one of a large scale numerical problem, but of the need to explore a larger parameter space. It is perhaps also questionable that a marked improvement on the flux estimates will be obtained, even if the estimate on the variational problem is replaced by its true solution.

In another study, Prof. R. Krishnamurti and Dr. H. Yang have completed their investigation of finite amplitude Hele-Shaw convection with an imposed shear. Their interests were also to determine the effect of convective transport of momentum upon the mean flow. An imposed shear flow upon a horizontal layer of fluid, convecting at low Rayleigh numbers, leads to an alignment of rolls with their vorticity vectors along the imposed flow ("longitudinal rolls"). The Hele-Shaw geometry, on the other hand, forces the important constraint that convective cells have their vorticity vectors orthogonal to the imposed Couette or Poiseuille flow ("transverse rolls"). It is only in this latter arrangement that vertical transport of horizontal momentum is found to be counter to the imposed gradient. The convective transport thus tends to enhance the imposed shear.

The Hele-Shaw geometry has the additional advantage that the flow is two-dimensional. Thus Drs. Yang and Krishnamurti were able to solve the Boussinesq equations (rather than truncated model equations as in Howard and Krishnamurti, 1986) by using a modified perturbation expansion in powers of the amplitude ϵ , and a regular perturbation expansion in the powers of the Peclet number representing the imposed shear flow.

This work is represented in the *Ph.D. dissertation of H. Yang*, and two papers by Yang and Krishnamurti, one of which was recently submitted for publication.

LOW DIMENSIONAL BAROTROPIC PROBLEMS

Prof. S. Blumsack has continued his analysis of approximate solutions of the barotropic vorticity equation $\nabla^2 \psi_t - \psi_y \nabla^2 \psi_x + \psi_x \nabla^2 \psi_y = 0$, in the square $0 < x < 2\pi$, $0 < y < 2\pi$, subject to periodic boundary conditions. Highly-truncated doubly periodic series of the form $\psi(x, y) = \sum_{k,l} a_{kl}(t) e^{i(kx+ly)}$ were assumed, resulting in a system of ordinary differential equations for the (complex) coefficients $a_{kl}(t)$. This approach has been undertaken to help check and understand numerical models.

Special attention was devoted to a particular truncation—a rhomboidal truncation for which $||k| - |l|| \leq 2$ and $(k, l) \neq (0, 0)$. The resulting dynamical system contains twelve degrees of freedom. A six parameter family of equilibrium states was found. Two of the parameters account for the fact that any equilibrium state can be translated arbitrarily in the x - y plane to become another equilibrium state. The other four parameters can be taken to be the magnitudes of the Fourier

modes.

The time-dependence for this twelve mode truncation has been investigated in detail. The simplified dynamics require that the amplitudes for modes $(k, l) = (0, \pm 2), (\pm 2, 0)$ be constant in time. Conservation of energy and enstrophy then imply that the total energy for the subset of modes $\{(k, l) = (1, \pm 1), (-1, \pm 1)\}$ is constant. In fact, this four-dimensional subspace splits into a pair of two-dimensional subspaces, with energy conserved in each. A similar argument results in energy conservation for the subset $\{(k, l) = (\pm 1, 0), (0, \pm 1)\}$. Trajectories in this four-dimensional subspace are generally complicated, but several special cases do result in circular trajectories in certain planes. These cases have been analyzed in detail.

Most of the issues related to the invariances and time dependence for the twelve mode model have been dealt with. The next level model in terms of complexity has twenty-four degrees of freedom; a rectangular truncation containing all modes such that $|k| \leq 2, |l| \leq 2$ (but again $(k, l) \neq (0, 0)$). Once again there exists a six parameter family of equilibrium states. Two of these are associated with the translational invariance of the problem and one of them is associated with the fact that if ψ_0 is an equilibrium solution, then so is any multiple of ψ_0 .

Effort continued in the analysis of the time-dependent problem. The model was linearized about an arbitrary equilibrium state and the resulting matrix computed. The fact that the equilibrium state can be represented by means of six parameters implies that six (out of the twenty-four) eigenvalues must be zero. Eigenvalues having non-zero values for both real and imaginary parts come in clusters of four of the form $\pm a \pm ib$; there can be at most four of these clusters since there are only eighteen non-zero eigenvalues. Limited numerical experimentation indicated two such clusters plus five pair of complex conjugate, purely imaginary eigenvalues.

The eigenvalue structure suggests that the dynamical system is unlikely to have heteroclinic orbits (that do not consist entirely of equilibrium points). At most nine trajectories leave and at most nine approach any equilibrium point. There seems to be too much room in our twenty-four dimensional space to permit a departing trajectory to meet with a trajectory incoming at a different equilibrium point.

In another study, Prof. R. L. Pfeffer and student Wen Ding have made considerable progress solving the nonlinear steady state barotropic vorticity equation for parameters which correspond

to a series of laboratory experiments involving forced flow of a rotating fluid over bottom topography. In these experiments, conducted in our Institute, the fluid is contained in a rotating circular cylindrical annulus with a differentially rotating, rigid, radially sloping, lid in contact with the top surface of the fluid. The fluid is forced into motion by the rotation of the lid. The governing equation for the steady state flow that develops when we are far from resonance is

$$J(\psi, \epsilon \nabla^2 \psi + h - \beta R) + E^{1/2} \nabla^2 \psi - E \nabla^4 \psi = E^{1/2} \quad (1)$$

\uparrow
vorticity
advection

\uparrow
top
surface
slope
(β -effect)

\uparrow
internal
diffusion

\uparrow
forcing by
differentially
rotating lid

\uparrow
topographic
forcing

\uparrow
Ekman
boundary
layer
dissipation

The side wall boundary conditions are $\partial\psi/\partial R = \partial\psi/\partial\lambda = 0$. Here, ψ is the dimensionless stream function, ϵ is the Rossby number (ratio of inertial to Coriolis acceleration), h is the ratio of the topographic height to the mean fluid depth, β is a dimensionless parameter proportional to the radial slope of the rigid lid (which simulates the variation of the Coriolis parameter with the latitude on the earth), R is radius from the axis of rotation and E is the Ekman number (ratio of viscous force to Coriolis acceleration).

Dr. Pfeffer and Mr. Ding have compared solutions using these different methods to determine an acceptable level of truncation. The idea is to determine the lowest order nonlinear system that can account for the observed fluid behavior. By studying the solutions of such a system they hope to gain insight into the physical mechanisms involved in different parameter ranges and to set the stage for deriving a low order dynamical system to account for time-dependent behavior of related baroclinic flows. The three methods used to solve this problem are finite difference, semi-spectral and Chebyshev collocation. In the finite difference method they solved the problem iteratively using a grid of 31 points in the radial direction and 61 in the azimuthal direction. In the semi-spectral approach they used 31 grid points in the radial direction and 5 azimuthal wave numbers. And in the Chebyshev collocation method they used 7 Chebyshev polynomials to represent the radial variation of the stream function and 5 Fourier components to represent the azimuthal variation.

Figs. 1a, b and c show the solutions for the phase and amplitude of azimuthal wave number two (the topographic wave number) and the azimuthally-averaged azimuthal velocity \bar{U} , respectively, as a function of the rotation rate Ω of the system. The solutions are almost indistinguishable from one another. Figs. 1d, e and f show experimentally determined values of the same parameters at five different rotation rates against a backdrop of the finite difference solution. The quantitative behavior of the solution is similar to that of the experimental data. There are, however, significant quantitative discrepancies that suggest reexamination of certain assumptions inherent in the theory as expressed by Eq. (1). One such assumption is that the effect of topography and that of bottom and top boundary layer dissipation are completely separable as they are for small slopes. For large slopes, certain correction terms come into play which will have to be taken into account. Another assumption is that the side wall boundary layers can be neglected. This becomes increasingly invalid as the rotation rate becomes large. In the coming year, Dr. Pfeffer and Mr. Ding plan to incorporate the appropriate corrections in an effort to bring the theory into full compliance with the experimental data.

In spite of the obvious quantitative differences between theory and experiment, the qualitative similarities, and the fact that they can be represented well by a low order model in terms of a truncated series of Fourier components in the azimuthal direction and Chebyshev polynomials in the radial direction, suggests that it should be possible to use the low order model to shed light on the relative roles of the β -effect and dissipation in bringing about the variations of phase, amplitude and \bar{U} with Ω shown in Fig. 1. This will be the focus of Dr. Pfeffer's and Mr. Ding's efforts in the coming year.

Some questions that have arisen in connection with Dr. Pfeffer's and Mr. Ding's work on this problem have led Prof. Howard to look more closely into the derivation of approximate equations and their boundary conditions for rotating, nearly geostrophic, flow in containers of arbitrary shape and with rather general boundary conditions. He had previously considered such questions in some detail (also for stratified flow (L. N. Howard, *Lectures in Applied Math.*, 13, 121-124, 1971), but only in the linear context. His conjecture is that the non-linear quasi-geostrophic system, though it has been extensively studied and applied, has not yet been quite properly fitted into a general formulation. This investigation is not finished yet, but has reached a stage where a preliminary

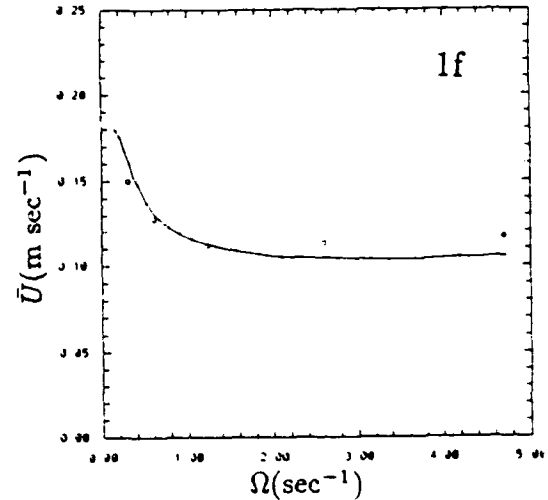
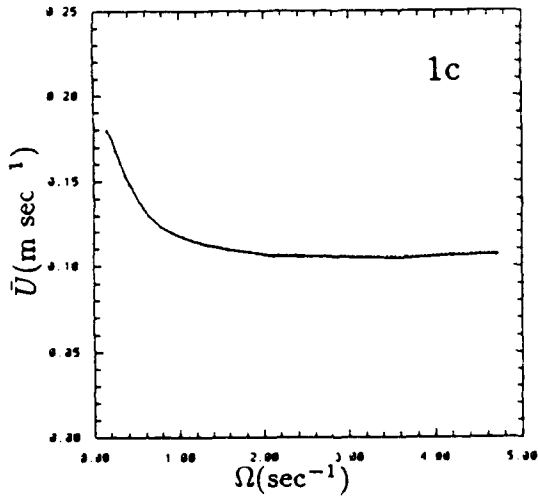
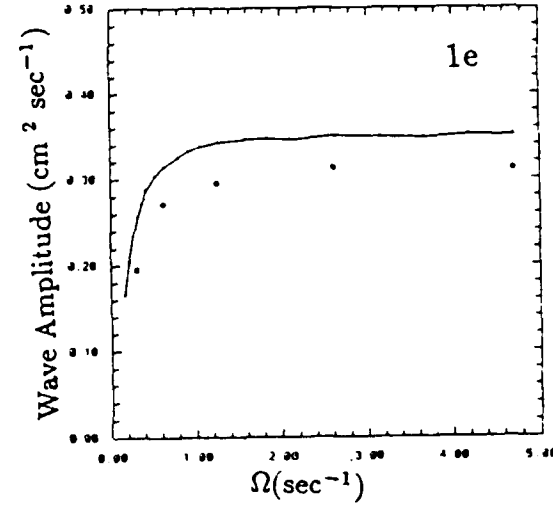
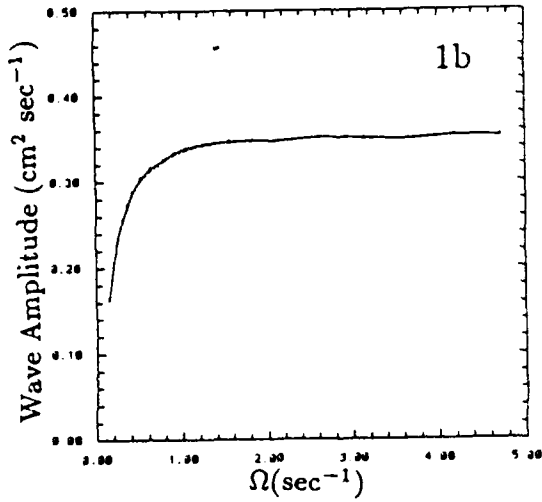
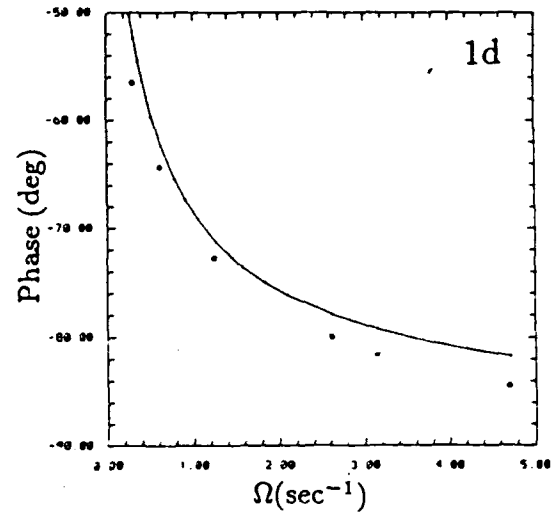
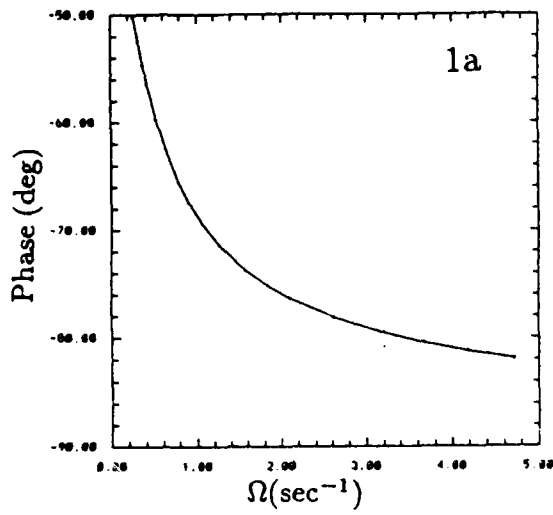


Fig. 1. Solutions of the barotropic vorticity equation by three different methods for a. phase vs Ω , b. amplitude vs Ω and c. \bar{U} vs Ω . The slight discrepancies disappear completely when we take more grid points in the azimuthal direction in the finite difference solution.

Comparisons between the finite difference solution and experimental determinations of d. phase, e. amplitude and c. azimuthal mean velocity \bar{U} .

attempt at writing it up seems desirable, and Dr. Howard plans to do this in the coming year. The basic structure of the formulation can be described briefly as follows.

When the Rossby and Ekman numbers are small, and time scales are long compared to the rotation period, the interior flow approximately satisfies the (homogeneous) linear geostrophic equations, with approximately zero normal velocity. However, the homogeneous geostrophic equations, with zero normal velocity, have in general many solutions; in the simplest case where the "depth" of the container—measured parallel to the rotation axis—is constant, the pressure is essentially an arbitrary function of the transverse variables (and determines the velocity components). When the depth is not constant, the arbitrariness is less extreme (the geostrophic flow must be along contours of constant depth), but still present. As with all linear problems, when the homogeneous form has many solutions, the non-homogeneous equations (and boundary conditions) can have solutions only when the non-homogeneities satisfy certain "solvability conditions." These conditions are sometimes expressed as conditions of orthogonality to all solutions of the adjoint homogeneous problem; in the present case the adjoint homogeneous problem has many solutions, depending in part on the nature of the container, and the solvability conditions are somewhat complicated in general. The full equations can be regarded as the geostrophic equations with non-homogeneities given by the terms involving the Rossby and Ekman numbers. The boundary conditions on the interior flow come from the imposed boundary conditions (differential motions of the boundary, sources and sinks, etc.) modified by the effects of boundary layers (whose structure is in general also coupled to the interior flow) and also appear as inhomogeneities in the geostrophic problem. The solvability conditions then lead to additional relations on the interior flow (like the potential vorticity equation and the Ekman layer matching—these relations are not necessarily linear), which ultimately determine which of the many geostrophic flows is in fact the first approximation to the real flow, and what its higher corrections are. This general description is fairly straightforward, but working it out in detail is rather complex.

In another effort, Professors Barcilon and Blumsack and Mr. Y. Chang are investigating the dynamics of equilibrium states in a sheared barotropic channel flow. This flow is relevant to meteorological flows and the object of this work is to understand the population of certain states, called free modes (see below), and comprehend how the actual flow in phase space is attracted from

one state to another. Thus, this problem has some bearing on atmospheric predictability and was strongly motivated by the work of Branstator and Opsteegh (*J. Atmos. Sci.*, **46**, 1799–1814, 1989).

Thus, they are interested in finding the free states of the barotropic vorticity equation for a given shear flow. These states are characterized by the equation

$$J(\Psi, \nabla^2 \Psi) = 0$$

where Ψ is the streamfunction and $\nabla^2 \Psi$ the vorticity.

Following Vallis et al. (*J. Fluid Mech.*, **207**, 133–152, 1989) they have developed a numerical scheme which locates these states starting from a given initial condition. The property of this scheme is that it conserves the vorticity of a given particle but allows the total energy of the system to be a monotonic function of time, the total energy approaching a constant when a free state is reached.

Research has been pursued along two avenues:

1. They determined possible steady wavy solutions of the barotropic vorticity equation. These solutions involve sheared flows constructed by means of trigonometric functions.
2. They chose as an initial basic state

$$\bar{u}_0(y) = -u_0 \tanh(y)$$

superimposed an initial perturbation of the form

$$\psi(x, y, 0) = A_0 \sin \ell_0(y + \pi) \sin k_0 x$$

and integrated the vorticity equation using a spectral method. Various initial conditions were considered by specifying different values of the zonal wave number k_0 (the most unstable $k_0 = 0.44$ by linear theory) as well as different numbers of modes. They then compared these dynamics with the dynamics of the packages that search for the free states of the system.

A short note entitled “Some Preliminary Results on Equilibrium States in a Sheared Barotropic Flow” by Y. Chang, A. Barcilon and S. Blumsack was sent earlier to AFOSR. The authors plan to expand this note into a paper which will be submitted for publication before the end of the year.

NUMERICAL METHODS

The shallow water equations are a set of first order nonlinear hyperbolic partial differential equations having many important applications in meteorology and oceanography. These equations can be used in studies of tides and surface water run-off. They may also be used to study large-scale waves in the atmosphere and ocean if terms representing the effects of the earth's rotation (Coriolis terms) are included. Indeed it has become customary, in developing new numerical methods for numerical weather prediction or oceanography, to study first the simpler nonlinear shallow water equations system, which possesses the same mixture of slow and fast-moving waves as the more complex baroclinic three-dimensional primitive equations of motion. Galerkin finite element methods have been successfully applied to these equations by many researchers. However, these algorithms were not designed to run efficiently on parallel processing architectures.

In recent years there was a great interest in parallel computations due to the advent and growing popularity of parallel computers. With an appropriate design of the algorithm, all the processors within a single computer can be called upon to work on a single program concurrently and thus the result can be obtained in a much shorter period of time. A parallel computer with n processors is cheaper in CPU time than n computers each having a single processor. Parallel computers can be used to tackle such problems as worldwide weather forecasting and the like which constitute a complex task for serial machines. By carrying out research in parallel processing, research workers may further explore the potential power of computer resources. Parallel processing makes it possible to attain the goal of full utilization of the most powerful high-performance supercomputers.

In view of the importance of the shallow water equations in meteorology and oceanography and the obvious advantages offered by modern parallel computers, the goal in the present research effort is to investigate new parallel computing techniques and develop fast algorithms for the efficient utilization of parallel computers in the solution of the aforementioned set of partial differential equations. The expected results of this research will impact on both the area of parallel scientific computation as well as on application problems in meteorology and oceanography.

Parallelism present in the numerical solution algorithms of PDE's can be exploited at several levels. Domain decomposition is the highest level of parallelism in the sense that the whole problem is split up into several smaller sub-problems which will be apportioned among, and handled by,

several processors concurrently even before we discretize it to obtain the appropriate linear or nonlinear systems of algebraic equations. This technique has been chosen as the general methodology of Prof. I. M. Navon and student Yihong Cai to be employed in mapping the whole computational effort into a number of processors for parallel processing.

Under the present URI, they have successfully decomposed the limited area domain, over which the finite-element model of the hyperbolic nonlinear shallow water equations are defined, into two subdomains and then into four subdomains by applying a Schur complement domain decomposition technique, which is a nonoverlapping domain decomposition approach. The results were more satisfactory than expected in the sense that a speed-up had already been achieved even on a single processor. They have also formulated the framework of the general technique which can be applied to an arbitrary number of subdomains.

One of the key steps for carrying out the Schur domain decomposition is to obtain the arrow-head shaped matrix, from which information can be derived regarding the Schur matrix complement and the systems governing the various subdomain problems. To build a device into their program which allows for the easy modification of the code to deal with different resolutions for obtaining higher and higher orders of accuracy and testing for the convergence of the method, Dr. Navon and Mr. Cai have found and devised an efficient way, namely a renumbering algorithm, for transforming the original system matrix into the arrow-head matrix. This renumbering algorithm allows for an easier implementation designed to carry out the Schur complement domain decomposition and also provides an easy means by which to efficiently implement multicoloring techniques.

Another very important issue in the Schur complement domain decomposition is how to avoid the explicit evaluation of the Schur complement. It turns out that is very expensive to obtain the Schur complement as it requires kn_i solves, where n_i is the number of nodes in the interface and k is the number of subdomains.

The explicit computation of the Schur complement will probably dominate the overall solution process for large degrees of freedom on the interface as the mesh is refined. To apply such acceleration methods as Conjugate Gradient Squared (CGS) or Generalized Minimal Residual (GMRES) to the nonsymmetric matrix interface system, Dr. Navon and Mr. Cai have formulated an algorithm which avoids the explicit calculation of Schur complement. This algorithm can be described as the

inner loop across the subdomains and outer loop on the interface. They are now at the stage where they are ready to carry out extensive numerical experiments based on this idea and the results will be published in the coming months.

The interface system is usually solved by an iterative technique and each iteration for the interface requires solving all the subdomain problems once. It is therefore of paramount importance to make the number of iterations on the interface as small as possible. In order to achieve this, various interface preconditioning techniques will be investigated in the coming year so as to change the eigen-spectrum properties in order to reduce the condition number of the problem as well as to cluster the eigenvalues in the spectrum and thus speed up the iteration process.

Both modified incomplete LU decomposition preconditioning methods as well as boundary probe methods will be considered as prime candidates. This work is important since, if an optimal preconditioner can be found for either the interface system or the subdomain nonsymmetric matrix systems, the time required for calculation will be dramatically reduced. This is because there will result a significant reduction in the number of iterations required to achieve a pre-specified convergence criterion. Since the solution of the interface system is the overhead for parallel processing, they will also study the possibilities of parallelizing both the acceleration methods as well as the preconditioners. A further speed-up can be expected from this research.

An additional effort will be dedicated towards parallelizing the various phases in the finite element discretization algorithm such as the assembly procedure, node numbering and calculation of the element matrices. The computational advantage of exploiting this type of parallelism has already attracted the attention of many researchers. However, there is still a sizable amount of research to be carried out for obtaining adequate parallel algorithms. These parallel algorithms will be tested on various computer architectures such as the Cray Y-MP supercomputer, hypercube machines and Alliant parallel computers.

So far, Dr. Navon and Mr. Cai we have carried out the domain decomposition using the nonoverlapping approach. However, there is another domain decomposition approach called the overlapping approach. This consists in a broad class of techniques in which initial guesses are required for the overlapped subdomain regions. The solution involves an iterative process to be carried out until the difference between different subdomains over the same overlapped region achieves a prescribed

tolerance. They plan to use the theory of optimal control to address this problem in the future. This will require them to develop the adjoint model of the finite-element discrete version of the shallow-water equations and to use a large scale unconstrained minimization algorithm to minimize a least squares cost functional measuring the lack of fit between two overlapped subdomain solutions.

BAROCLINIC INSTABILITY

This section deals with our progress toward understanding the development and nonlinear behavior of traveling baroclinic waves in the atmosphere and their interaction with topographically and thermally forced planetary waves.

In a paper entitled "Effects of Wave-Wave and Wave-Mean Flow Interactions on the Evolution of a Baroclinic Wave" (*Geophys. Astrophys. Fluid Dyn.*, 56, 59-79, 1991), Professors A. Barcilon and T. Nathan (visitor from the University of California at Davis) examined within the context of a two-layer model the weakly nonlinear evolution of a free baroclinic wave in the presence of slightly supercritical, vertically sheared zonal flow and a forced stationary wave field that consists of a single zonal scale and an arbitrary number of meridional harmonics. The presence of the (planetary-scale) stationary wave introduces zonal variations in the supercriticality and is shown to alter the growth rate and asymptotic equilibrium of the (synoptic-scale) baroclinic wave via two distinct mechanisms: The first is due to the direct interaction of the stationary wave with that portion of the mean field that is corrected by the zonally rectified stationary wave fluxes (wave-mean mechanism). These mechanisms can oppose or augment each other depending on the amplitude and spatial structure of the stationary wave field. If the stationary wave field is confined primarily to the upper (lower) layer and consists only of the gravest cross-stream mode, conditions are favorable (unfavorable), for nonzero equilibrium of the free wave.

In addition to the time-dependent heat flux generated by baroclinic growth of the free wave, its interaction with a stationary wave field consisting of two or more meridional harmonics generates time-dependent heat fluxes that vary with the period of the free wave. However, if the stationary wave field contains several meridional harmonics of sufficiently large amplitude, the free baroclinic wave is destroyed.

In another paper entitled "Asymmetric Ekman Dissipation, Sloping Boundaries and Linear Baroclinic Instability" (*Geophys. Astrophys. Fluid Dyn.*, 54, 1991, in press) Drs. H.-Y. Weng and A. Barcilon examined the effects of symmetric and asymmetric Ekman dissipation on baroclinic instability, phase speed and wave structure in a linear Eady-like channel model with/without oppositely sloping boundaries. In the absence of sloping boundaries, symmetric Ekman dissipation has a stabilizing tendency for all waves. When the Ekman layers are asymmetric, the viscous asymmetry destabilizes waves by (i) extending the unstable waveband toward both the long and short waves, and (ii) increasing their growth rates, compared with the viscous symmetric case. When the asymmetric dissipation is very small, the destabilization may result in a frictional instability for short waves which are stable in the inviscid case. The viscous asymmetry makes waves dispersive and their structure asymmetric about mid-depth. However, the *sense* of the viscous asymmetry and the *sign* of shear do not affect the instability, but do modify the direction of phase propagation and the shape of the wave structure.

With asymmetric Ekman dissipation and sloping boundaries, frictional instability is a mixture of three mechanisms: (i) symmetric dissipation in the presence of sloping boundaries, (ii) viscous asymmetry in the absence of sloping boundaries; and (iii) the sense of viscous asymmetry in the presence of sloping boundaries, which is sensitive to the wavenumber and the sign of shear. The slope renders the phase speed more dispersive and sensitive to the sign of shear and the sense of the viscous asymmetry. For westerly shear, the slope reduces (increases) the wave amplitude and makes the wave more baroclinic (barotropic) in the lower (upper) level. For a given wavenumber, easterly shear dynamics may be deduced from westerly shear dynamics by a proper change of the sense of the viscous asymmetry.

In another ongoing project under the URI, one of our regular visitors, Prof. T. Nathan of U. C. Davis, is examining the response of a multiwave baroclinic wave system to periodic (seasonal) variations in zonal mean thermal forcing. The goal is to understand the role of nonlinearity in the non-cyclic response of the earth's atmosphere (which exhibits strong differences in circulation and climate from year to year) to periodic forcing by the sun. In view of the fact that even the simplest dynamical system can exhibit a wide range of spatial and temporal variability when subjected to periodic forcing, the following question naturally arises: to what extent are the pronounced

differences in weather phenomena of the same season of different years due to seasonal forcing? Addressing this question forms the basis of Dr. Nathan's research under the URI.

The model is basically that of Nathan (*J. Atmos. Sci.*, 45, 1052-1071, 1988; *J. Atmos. Sci.*, 45, 113-130, 1989), modified to account for periodic variations in zonal mean potential vorticity. Briefly, he considers the classic two-layer model for flow confined to a zonally periodic, midlatitude beta-plane channel. The flow is forced by sinusoidal bottom topography and periodic variations in zonal mean potential vorticity. Potential vorticity damping is the sole dissipative mechanism.

The basic state flow is zonal, steady, and independent of latitudinal variations. Dr. Nathan considers a disturbance field consisting of a free marginally unstable baroclinic wave and a resonant topographic wave. The analysis hinges on introducing "long" time scales and expanding the disturbance field in an asymptotic series. The problem then reduces to a sequence of inhomogeneous linear differential equations where the nonlinear terms appear as forcing terms on the linear operator. To insure the validity of the asymptotic expansions, one must remove those terms which are orthogonal to the homogeneous adjoint solution. This solvability condition yields the sought after temporal evolution equations for the wave amplitudes and mean field corrections.

The equations governing the temporal evolution of the free unstable wave amplitude, $A(t)$, forced wave amplitude, $B(t)$ and mean field corrections, $\Phi_1(y, t)$ and $\Phi_2(y, t)$, have been derived and can be written in general form as:

$$\begin{aligned}
& \frac{d^2 A}{dt^2} - [a_1 + a_2 h^2] A + 2r \frac{dA}{dt} + i[a_3 + a_4 h^2] \frac{dA}{dt} + a_5 A |B^2| + a_6 (B + B^*) h A \\
& + i a_8 \frac{dA}{dt} |B^2| + i a_9 (B + B^*) h \frac{dA}{dt} \\
& - i \frac{dA}{dt} \int_0^1 \left\{ \frac{\partial}{\partial y} [a_{10} M_1 + a_{11} \Phi_1 + a_{12} M_2 + a_{13} \Phi_2] \right. \\
& \quad \left. + a_{14} \Phi_1 + a_{15} M_2 + a_{16} \Phi_2 \right\} \sin^2 \pi y dy \\
& - A \frac{\partial}{\partial t} \int_0^1 \frac{\partial}{\partial y} \{a_{10} M_1 + a_{11} \Phi_1 + a_{12} M_2 + a_{13} \Phi_2\} \sin^2 \pi y dy \\
& + A \int_0^1 \frac{\partial}{\partial y} \{a_{17} M_1 + a_{18} \Phi_1 + a_{18} M_2 + a_{19} \Phi_2\} \sin^2 \pi y dy = 0 \tag{1}
\end{aligned}$$

$$\frac{dB}{dt} - i b_1 B + r B + i B \int_0^1 \frac{\partial}{\partial y} \{b_2 M_1 + b_3 \Phi_1 + b_4 \Phi_2\} \sin^2 \pi y$$

$$+ ib_5 h \int_0^1 \frac{\partial \Phi}{\partial y} \sin^2 \pi y dy + ib_6 B |A|^2 + ib_7 h |A|^2 = 0 \quad (2)$$

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \tau \right) M_j &= (-1)^j \left(c_1 \frac{d|A|^2}{dt} + c_2 \frac{d|B|^2}{dt} + c_3 \tau |A|^2 + c_4 \tau |B|^2 \right) \\ &+ (-1)^j c_5 h |A|^2 (iB + *) \sin \pi y \\ &+ h \left(c_6 \frac{dB}{dt} + \tau B \right) \sin^2 \pi y + * + H \sin(t/\tau); \quad j = 1, 2. \end{aligned} \quad (3)$$

where,

$$M_j = \frac{\partial^2 \Phi_j}{\partial y^2} + (-1)^j F(\Phi_1 - \Phi_2) \quad (4)$$

is the zonal mean potential vorticity correction. In the above equations a_p ($p = 1-19$), b_m ($m = 1-9$), and c_n ($n = 1-6$) are constraints that depend on zonal wavenumbers of the free and forced waves, internal rotational Froude number, F , planetary vorticity factor, β , and basic state zonal wind in the upper layer, U_1 . The topographic height is given by h , τ measures the strength of the potential vorticity damping, the asterisk represents a complex conjugation, $i = \sqrt{-1}$, H is the amplitude of the zonal mean periodic forcing, and τ is the period of the zonal mean forcing.

To cast Eqs. (1)–(3) in a form suitable for numerical integration, the mean field corrections were written as

$$\Phi_j(y, t) = \sum_{m=1}^{N_T} \Psi_m^{(j)}(t) \cos m\pi y; \quad j = 1, 2. \quad (5)$$

where the terms with m odd are zero. Insertion of (5) into (1)–(4) yields a system of coupled, nonlinear, ordinary differential equations, which were integrated forward in time using a fourth-order scheme with a nondimensional time step of 0.1. For the majority of integrations, $N_T = 12$ proved sufficient. The following initial conditions were used: $\Psi_M^{(j)} = B = dA/dt = 0$ at $t = 0$.

In the absence of periodic (seasonal) forcing, numerical integrations of the evolution equations performed under the URI thus far reveal two distinct asymptotic states of the system: one is a single (stationary) topographic wave state, or a mixed wave state where the baroclinic wave propagates at a fixed amplitude while the topographic wave remains stationary and phase shifted with respect to the topography. The single topographic wave states are, relatively speaking, favored for large zonal scales, large topographic heights and weak dissipation. However, for sufficiently weak dissipation, inclusion of zonal mean periodic forcing produces quasi-periodic, mixed wave regimes

which correspond to "interannual" variations in the model circulation. Wave-wave interactions have been shown to play a vital role in determining the "interannual" variability of the model circulation. In particular, wave-wave interactions are responsible for the following: i) the interchange of wave dominance from one "winter" ("summer") to the next (when the system exhibits nonperiodic responses); ii) reducing some sub-cycle responses to a periodic response with the period of the forcing; and iii) confining the topographic wave to a more narrow region of phase space than would otherwise occur in the absence of wave-wave interactions.

During the next year Dr. Nathan plans to extend the above model to include zonally asymmetric periodic forcing and simulate the influence of seasonal changes in land-sea heating contrasts.

MOUNTAIN WAVES

Other research which has developed in the course of the present URI has led to an additional paper entitled "Reflection of Hydrostatic Mountain Waves from Spatially Nonuniform Layers" (*Geophys. Astrophys. Fluid Dyn.*, 1991, in press). In this work, Professor A. Barcilon has collaborated with Prof. W. Blumen of the University of Colorado in an investigation of the steady, hydrostatic, inviscid, Boussinesq flow of a stably stratified fluid over a bell-shaped ridge within the framework of a linear model. The three layer model atmosphere introduced is such that the Brunt-Väisälä frequency is constant in each layer but the interfaces of the middle layer are allowed to vary gently in the cross-ridge direction. In essence, the model can be tuned in both *vertical* and *horizontal* directions. These cross-ridge variations can produce significant differences in both the cross-ridge surface wind and the surface drag compared to the response obtained by use of a horizontally uniform reflecting layer. These changes are sensitive to both the vertical location of the middle layer and to the slope of its lower interface at the ridge crest. Many of these features are explained by means of a conventional layered-model analysis.